Solution 17 :

1°/ $\Delta = (\sqrt{3}-3i)^2 - 4\times 2\times(-4) = 3 - 6\sqrt{3}\,i - 9 + 32 = 26 - 6\sqrt{3}\,i$

Cherchons x et y tel que : $(x+iy)^2 = \Delta$

$$\begin{cases} x^2 - y^2 = 26 \\ x^2 + y^2 = \sqrt{(26)^2 + (6\sqrt{3})^2} \\ 2xy = -6\sqrt{3} \end{cases} \Leftrightarrow \begin{cases} x^2 - y^2 = 26 \quad (1) \\ x^2 + y^2 = 28 \quad (2) \\ xy = -3\sqrt{3} \end{cases}$$

$$(1) \text{ et } (2) \Rightarrow \begin{cases} x^2 = 27 \\ y^2 = 1 \\ xy < 0 \end{cases} \Rightarrow \begin{cases} x = \pm 3\sqrt{3} \\ y = \pm 1 \\ xy < 0 \end{cases} \quad \text{donc} \quad \Delta = (3\sqrt{3} - i)^2$$

$$z' = \frac{(-\sqrt{3}+3i) - (3\sqrt{3} - i)}{4} = -\sqrt{3} + i$$

$$z'' = \frac{(-\sqrt{3}+3i) + (3\sqrt{3} - i)}{4} = \frac{\sqrt{3}}{2} + \frac{i}{2}$$

2°/ a) $\left|-\sqrt{3}+i\right| = \sqrt{3+1} = 2$; $\left|\frac{\sqrt{3}}{2} + \frac{1}{2}i\right| = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

donc $z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$; $z_2 = -\sqrt{3} + i$

$$z_1 = [r_1, \theta_1] \text{ on a : } r_1 = 1 ; \begin{cases} \cos\theta_1 = \frac{\sqrt{3}}{2} \\ \sin\theta_1 = \frac{1}{2} \end{cases} \Rightarrow \theta_1 \equiv \frac{\pi}{6}(2\pi)$$

donc $z_1 = \left[1, \frac{\pi}{6}\right] = e^{i\frac{\pi}{6}}$

$$z_2 = [r_2, \theta_2] \text{ on a : } r_2 = 2 ; \begin{cases} \cos\theta_2 = \frac{-\sqrt{3}}{2} \\ \sin\theta_2 = \frac{1}{2} \end{cases} \Rightarrow \theta_2 \equiv \frac{5\pi}{6}(2\pi)$$

donc $z_2 = \left[2, \frac{5\pi}{6}\right] = 2e^{i\frac{5\pi}{6}}$